PROBLEMA RESUELTO 2

Calcule la derivada de la función

$$f(x) = \ln\left(\frac{x-2}{x+2}\right) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Solución

Utilizando las propiedades de los logaritmos para expresar el primer término como una suma de logaritmos

$$f(x) = \ln(x-2) - \ln(x+2) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Calculando la derivada

$$\begin{aligned} f'(x) &= D_x\left[\ln(x-2) - \ln(x+2) + \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right] \\ &= \frac{1}{x-2} - \frac{1}{x+2} + \sqrt{2}\cdot\frac{1}{1+\left(\frac{x}{\sqrt{2}}\right)^2}\cdot D_x\left(\frac{x}{\sqrt{2}}\right) \\ &= \frac{1}{x-2} - \frac{1}{x+2} + \sqrt{2}\cdot\frac{1}{1+\left(\frac{x}{\sqrt{2}}\right)^2}\cdot\left(\frac{1}{\sqrt{2}}\right) \end{aligned}$$

Simplificando

$$\begin{aligned} f'(x) &= \frac{1}{x-2} - \frac{1}{x+2} + \frac{1}{1+\frac{x^2}{2}} \\ &= \frac{1}{x-2} - \frac{1}{x+2} + \frac{2}{2+x^2} \\ &= \frac{(x+2)(2+x^2) - (x-2)(2+x^2) + (x-2)(x+2)}{(x-2)(x+2)(2+x^2)} \\ &= \frac{2x+4+x^3+2x^2-2x+4-x^3+2x^2+x^2-4}{(x-2)(x+2)(2+x^2)} \\ &= \frac{5x^2+4}{(x-2)(x+2)(2+x^2)} \end{aligned}$$