Ejercicios sobre identidades trigonométricas

En los ejercicios 1 a 6 escriba la expresión en términos de senos y cosenos, luego simplifíquela

1. $\operatorname{sen} x \cot x$

2. $\cos t \csc t$

3. $\dfrac{\csc x}{\sec x}$

4. $\cot^2 x - \csc^2 x$

5. $\dfrac{\csc\theta - \operatorname{sen}\theta}{\cos\theta}$

6. $\dfrac{\tan\alpha}{\sec\alpha - \cos\alpha}$

En los ejercicios 7 a 10 determine si la ecuación dada es una identidad. Si la ecuación es una identidad, verifíquela. Si la ecuación no es una identidad, evalúe la ecuación en algún número para mostrar que no es una identidad.

7. $(\operatorname{sen} x + \cos x)^2 = \operatorname{sen}^2 x + \cos^2 x$

8. $\tan^4 x = \sec^4 x + 1$

9. $\cot x \csc x \sec x = 1$

10. $\sec^2 x + \csc^2 x = 2 + \cot^2 x + \tan^2 x$

En los ejercicios 11 a 60 verifique la identidad trigonométrica

11. $\tan x \csc x \cos x = 1$

12. $\operatorname{sen} x \cot x \sec x = 1$

13. $\dfrac{4\operatorname{sen}^2 x - 1}{2\operatorname{sen} x + 1} = 2\operatorname{sen} x - 1$

14. $\dfrac{\operatorname{sen}^2 x - 2\operatorname{sen} x + 1}{\operatorname{sen} x - 1} = \operatorname{sen} x - 1$

15. $(\operatorname{sen} x - \cos x)(\operatorname{sen} x + \cos x) = 1 - 2\cos^2 x$

16. $\dfrac{\cos\theta}{1 - \operatorname{sen}\theta} = \sec\theta + \tan\theta$

17. $\dfrac{1 - \tan^4\theta}{\sec^4\theta} = 1 - \tan^2\theta$

18. $\dfrac{1 + \tan^3\theta}{1 + \tan\theta} = 1 - \tan\theta + \tan^2\theta$

19. $\dfrac{\operatorname{sen}\theta}{1 - \cos\theta} - \dfrac{\operatorname{sen}\theta}{1 + \cos\theta} = 2\cot\theta$

20. $\dfrac{\cos\theta}{1 + \operatorname{sen}\theta} = \sec\theta - \tan\theta$

21. $\sec t = \dfrac{\cot t + \tan t}{\csc t}$

22. $\dfrac{\cos x \tan x + 2\cos x - \tan x - 2}{\tan x + 2} = \cos x - 1$

23. $\sec x - \tan x = \dfrac{1 - \operatorname{sen} x}{\cos x}$

24. $\dfrac{1}{\tan^2 x} - \dfrac{1}{\cot^2 x} = \csc^2 x - \sec^2 x$

25. $\operatorname{sen} x \tan x = \sec x - \cos x$

26. $\dfrac{1}{1 - \operatorname{sen} x} - \dfrac{1}{1 + \operatorname{sen} x} = 2\tan x \sec x$

27. $\dfrac{1 + \operatorname{sen} x}{1 - \operatorname{sen} x} - \dfrac{1 - \operatorname{sen} x}{1 + \operatorname{sen} x} = 4\tan x \sec x$

28. $\dfrac{\sec x - 1}{\sec x + 1} - \dfrac{\sec x + 1}{\sec x - 1} = -4\csc x \cot x$

29. $\dfrac{1}{1 - \cos x} - \dfrac{1}{1 + \cos x} = 2\csc^2 x - 1$

30. $\dfrac{1 + \operatorname{sen} x}{\cos x} = \dfrac{\cos x}{1 - \operatorname{sen} x}$

31. $(\operatorname{sen} x + \cos x + 1)^2 = 2(\operatorname{sen} x + 1)(\cos x + 1)$

32. $\dfrac{\sec x + \tan x}{\sec x - \tan x} = \dfrac{(\operatorname{sen} x + 1)^2}{\cos^2 x}$

33. $\sec^2 x\left(1 - \operatorname{sen}^2 x\right) = 1$

34. $\dfrac{\cos x}{1 - \operatorname{sen} x} - \tan x = \sec x$

35. $\dfrac{\cot x + \csc x \cos x}{\cot x} = 2$

36. $2\cos^2 x(\cot^2 x + 1) = \csc^2 x + \cot^2 x - 1$

37. $\csc^2 x + \sec^2 x = \csc^2 x \sec^2 x$

38. $\csc x + \operatorname{sen} x = \dfrac{\cot x}{\sec x - \tan x} - \dfrac{\cos x}{\sec x + \tan x}$

39. $\dfrac{2 + \csc x}{\sec x} - 2\cos x = \cot x$

40. $\dfrac{\tan y \operatorname{sen} y}{\tan y + \operatorname{sen} y} = \dfrac{\tan y - \operatorname{sen} y}{\tan y \operatorname{sen} y}$

41. $\cot^2 y \sec^2 y = 1 + \cot^2 y$

42. $\dfrac{\sec x + \cot x \csc x}{\sec^2 x} = \csc x \cot x$

43. $\dfrac{1 + \operatorname{sen} x - \cos^2 x}{\cos x + \operatorname{sen} x \cos x} = \tan x$

44. $\dfrac{(\tan x - \cot x)^2}{\sec^2 x \csc^2 x} = 1 - 4\operatorname{sen}^2 x \cos^2 x$

45. $\dfrac{\operatorname{sen}^2 x}{\tan x - \operatorname{sen} x} = \dfrac{\operatorname{sen}^2 x \cos x}{1 - \cos x}$

46. $\dfrac{\operatorname{sen}^6 x - \cos^6 x}{2\operatorname{sen}^2 x - 1} = 1 - \operatorname{sen}^2 x \cos^2 x$

47. $\dfrac{\cos^3 x + \sec^3 x}{\cos x + \sec x} = \sec^2 x - \operatorname{sen}^2 x$

48. $\sec^2 x \cot^2 x - \cos^2 x \csc^2 x = 1$

49. $\operatorname{sen}^6 x + \cos^6 x = 1 - 3\operatorname{sen}^2 x \cos^2 x$

50. $\dfrac{1 - \cos x}{1 + \cos x} = (\csc x - \cot x)^2$

51. $(1 - \operatorname{sen} x + \cos x)^2 = 2(1 + \operatorname{sen} x)(1 + \cos x)$

52. $\dfrac{\operatorname{sen}\theta}{\cot\theta + \csc\theta} - \dfrac{\operatorname{sen}\theta}{\cot\theta - \csc\theta} = 2$

53. $\operatorname{sen}^4\theta - \cos^4\theta = 1 - \dfrac{2\cot^2\theta}{\csc^2\theta}$

54. $\dfrac{\sec x + \tan x}{\sec x - \tan x} = \dfrac{(\operatorname{sen} x + 1)^2}{\cos^2 x}$

55. $\dfrac{1 - \operatorname{sen} x + \cos x}{1 + \operatorname{sen} x + \cos x} = \dfrac{\cos x}{1 + \operatorname{sen} x}$

56. $\dfrac{1 - \tan x + \sec x}{1 + \tan x - \sec x} = \dfrac{1 + \sec x}{\tan x}$

57. $\dfrac{4\tan x \sec^2 x - 4\tan x - \sec^2 x + 1}{4\tan^3 x - \tan^2 x} = 1$

58. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$

59. $\dfrac{\cot x - \cot y}{1 + \cot x \cot y} = \dfrac{\tan y - \tan x}{\tan x \tan y + 1}$

60. $\operatorname{sen}^2 x + 2\cos^2 x + \cos^2 x \cot^2 x = \csc^2 x$